Joseph-Louis Lagrange

Born: 25 Jan 1736 in Turin, Sardinia-Piedmont (now Italy)
Died: 10 April 1813 in Paris, France

Joseph-Louis Lagrange is usually considered to be a French mathematician, but the Italian Encyclopaedia [40] refers to him as an Italian mathematician. They certainly have some justification in this claim since Lagrange was born in Turin and baptised in the name of Giuseppe Lodovico Lagrangia. Lagrange's father was Giuseppe Francesco Lodovico Lagrangia who was Treasurer of the Office of Public Works and Fortifications in Turin, while his mother Teresa Grosso was the only daughter of a medical doctor from Cambiano near Turin. Lagrange was the eldest of their 11 children but one of only two to live to adulthood.

Turin had been the capital of the duchy of Savoy, but became the capital of the kingdom of Sardinia in 1720, sixteen years before Lagrange's birth. Lagrange's family had French connections on his father's side, his great-grandfather being a French cavalry captain who left France to work for the Duke of Savoy. Lagrange always leant towards his French ancestry, for as a youth he would sign himself Lodovico LaGrange or Luigi Lagrange, using the French form of his family name.

Despite the fact that Lagrange's father held a position of some importance in the service of the king of Sardinia, the family were not wealthy since Lagrange's father had lost large sums of money in unsuccessful financial speculation. A career as a lawyer was planned out for Lagrange by his father, and certainly Lagrange seems to have accepted this willingly. He studied at the College of Turin and his favourite subject was classical Latin. At first he had no great enthusiasm for mathematics, finding Greek geometry rather dull.

Lagrange's interest in mathematics began when he read a copy of Halley's 1693 work on the use of algebra in optics. He was also attracted to physics by the excellent teaching of Beccaria at the College of Turin and he decided to make a career for himself in mathematics. Perhaps the world of mathematics has to thank Lagrange's father for his unsound financial speculation, for Lagrange later claimed:-

If I had been rich, I probably would not have devoted myself to mathematics.

He certainly did devote himself to mathematics, but largely he was self taught and did not have the benefit of studying with leading mathematicians. On 23 July 1754 he published his first mathematical work which took the form of a letter written in Italian to Giulio Fagnano. Perhaps most surprising was the name under which Lagrange wrote this paper, namely Luigi De la Grange Tournier. This work was no masterpiece and showed to some extent the fact that Lagrange was working alone without the advice of a mathematical supervisor. The paper draws an analogy between the binomial theorem and the successive derivatives of the product of functions.

Before writing the paper in Italian for publication, Lagrange had sent the results to Euler, who at this time was working in Berlin, in a letter written in Latin. The month after the paper was published, however, Lagrange found that the results appeared in correspondence between Johann Bernoulli and Leibniz. Lagrange was greatly upset by this discovery since he feared being branded a cheat who copied the results of others. However this less than outstanding beginning did nothing more than make Lagrange redouble his efforts to produce results of real merit in mathematics. He began working on the tautochrone, the curve on which a weighted particle will always arrive at a fixed point in the same time independent of its initial position. By the end of 1754 he had made some important discoveries on the tautochrone which would contribute substantially to the new subject of the calculus of variations (which mathematicians were beginning to study but which did not receive the name 'calculus of variations' before Euler called it that in 1766).

Lagrange sent Euler his results on the tautochrone containing his method of maxima and minima. His letter was written on 12 August 1755 and Euler replied on 6 September saying how impressed he was with Lagrange's new ideas. Although he was still only 19 years old, Lagrange was appointed professor of mathematics at the Royal Artillery School in Turin on 28 September 1755. It was well deserved for the young man had already shown the world of mathematics the originality of his thinking and the depth of his great talents.

In 1756 Lagrange sent Euler results that he had obtained on applying the calculus of variations to mechanics. These results generalised results which Euler had himself obtained and Euler consulted Maupertuis, the president of the >Berlin Academy, about this remarkable young mathematician. Not only was Lagrange an outstanding mathematician but he was also a strong advocate for the principle of least action so Maupertuis had no hesitation but to try to entice Lagrange to a position in Prussia. He arranged with Euler that he would let Lagrange know that the new position would be considerably more prestigious than the one he held in Turin. However, Lagrange did not seek greatness, he only wanted to be able to devote his time to mathematics, and so he shyly but politely refused the position.

Euler also proposed Lagrange for election to the Berlin Academy and he was duly elected on 2 September 1756. The following year Lagrange was a founding member of a scientific society in Turin, which was to become the Royal Academy of Sciences of Turin. One of the major roles of this new Society was to publish a scientific journal the *Mélanges de Turin* which published articles in French or Latin. Lagrange was a major contributor to the first volumes of the *Mélanges de Turin* volume 1 of which appeared in 1759, volume 2 in 1762 and volume 3 in 1766.

The papers by Lagrange which appear in these transactions cover a variety of topics. He published his beautiful results on the calculus of variations, and a short work on the calculus of probabilities. In a work on the foundations of dynamics, Lagrange based his development on the principle of least action and on kinetic energy.

In the *Mélanges de Turin* Lagrange also made a major study on the propagation of sound, making important contributions to the theory of vibrating strings. He had read extensively on this topic and he clearly had thought deeply on the works of Newton, Daniel Bernoulli, Taylor, Euler and d'Alembert. Lagrange used a discrete mass model for his vibrating string, which he took to consist of n masses joined by weightless strings. He solved the resulting system of $n+1$ differential equations, then let n tend to infinity to obtain the same functional solution as Euler had done. His different route to the solution, however, shows that he was looking for different methods than those of Euler, for whom Lagrange had the greatest respect.

In papers which were published in the third volume, Lagrange studied the integration of differential equations and made various applications to topics such as fluid mechanics (where he introduced the Lagrangian function). Also contained are methods to solve systems of linear differential equations which used the characteristic value of a linear substitution for the first time. Another problem to which he applied his methods was the study the orbits of Jupiter and Saturn.

The Académie des Sciences in Paris announced its prize competition for 1764 in 1762. The topic was on the libration of the Moon, that is the motion of the Moon which causes the face that it presents to the Earth to oscillate causing small changes in the position of the lunar features. Lagrange entered the competition, sending his entry to Paris in 1763 which arrived there not long before Lagrange himself. In November of that year he left Turin to make his first long journey, accompanying the Marquis Caraccioli, an ambassador from Naples who was moving from a post in Turin to one in London. Lagrange arrived in Paris shortly after his entry had been received but took ill while there and did not proceed to London with the ambassador. D'Alembert was upset that a mathematician as fine as Lagrange did not receive more honour. He wrote on his behalf [1]:-

Monsieur de la Grange, a young geometer from Turin, has been here for six weeks. He has become quite seriously ill and he needs, not financial aid, for the Marquis de Caraccioli directed upon leaving for England

that he should not lack for anything, but rather some signs of interest on the part of his native country ... In him Turin possesses a treasure whose worth it perhaps does not know.

Returning to Turin in early 1765, Lagrange entered, later that year, for the Académie des Sciences prize of 1766 on the orbits of the moons of Jupiter. D'Alembert, who had visited the Berlin Academy and was friendly with Frederick II of Prussia, arranged for Lagrange to be offered a position in the Berlin Academy. Despite no improvement in Lagrange's position in Turin, he again turned the offer down writing:-

It seems to me that Berlin would not be at all suitable for me while M Euler is there.

By March 1766 d'Alembert knew that Euler was returning to St Petersburg and wrote again to Lagrange to encourage him to accept a post in Berlin. Full details of the generous offer were sent to him by Frederick II in April, and Lagrange finally accepted. Leaving Turin in August, he visited d'Alembert in Paris, then Caraccioli in London before arriving in Berlin in October. Lagrange succeeded Euler as Director of Mathematics at the Berlin Academy on 6 November 1766.

Lagrange was greeted warmly by most members of the Academy and he soon became close friends with Lambert and Johann(III) Bernoulli. However, not everyone was pleased to see this young man in such a prestigious position, particularly Castillon who was 32 years older than Lagrange and considered that he should have been appointed as Director of Mathematics. Just under a year from the time he arrived in Berlin, Lagrange married his cousin Vittoria Conti. He wrote to d'Alembert:-

My wife, who is one of my cousins and who even lived for a long time with my family, is a very good housewife and has no pretensions at all.

They had no children, in fact Lagrange had told d'Alembert in this letter that he did not wish to have children.

Turin always regretted losing Lagrange and from time to time his return there was suggested, for example in 1774. However, for 20 years Lagrange worked at Berlin, producing a steady stream of top quality papers and regularly winning the prize from the Académie des Sciences of Paris. He shared the 1772 prize on the three body problem with Euler, won the prize for 1774, another one on the motion of the moon, and he won the 1780 prize on perturbations of the orbits of comets by the planets.

His work in Berlin covered many topics: astronomy, the stability of the solar system, mechanics, dynamics, fluid mechanics, probability, and the foundations of the calculus. He also worked on number theory proving in 1770 that every positive integer is the sum of four squares. In 1771 he proved Wilson's theorem (first stated without proof by Waring) that n is prime if and only if $(n-1)! + 1$ is divisible by n. In 1770 he also presented his important work *Réflexions sur la résolution algébrique des équations* which made a fundamental investigation of why equations of degrees up to 4 could be solved by radicals. The paper is the first to consider the roots of an equation as abstract quantities rather than having numerical values. He studied permutations of the roots and, although he does not compose permutations in the paper, it can be considered as a first step in the development of group theory continued by Ruffini, Galois and Cauchy.

Although Lagrange had made numerous major contributions to mechanics, he had not produced a comprehensive work. He decided to write a definitive work incorporating his contributions and wrote to Laplace on 15 September 1782:-

I have almost completed a Traité de mécanique analytique, based uniquely on the principle of virtual velocities; but, as I do not yet know when or where I shall be able to have it printed, I am not rushing to put the finishing touches to it.

Caraccioli, who was by now in Sicily, would have liked to see Lagrange return to Italy and he arranged for an offer to be made to him by the court of Naples in 1781. Offered the post of Director of Philosophy of the Naples

Academy, Lagrange turned it down for he only wanted peace to do mathematics and the position in Berlin offered him the ideal conditions. During his years in Berlin his health was rather poor on many occasions, and that of his wife was even worse. She died in 1783 after years of illness and Lagrange was very depressed. Three years later Frederick II died and Lagrange's position in Berlin became a less happy one. Many Italian States saw their chance and attempts were made to entice him back to Italy.

The offer which was most attractive to Lagrange, however, came not from Italy but from Paris and included a clause which meant that Lagrange had no teaching. On 18 May 1787 he left Berlin to become a member of the Académie des Sciences in Paris, where he remained for the rest of his career. Lagrange survived the French Revolution while others did not and this may to some extent be due to his attitude which he had expressed many years before when he wrote:-

I believe that, in general, one of the first principles of every wise man is to conform strictly to the laws of the country in which he is living, even when they are unreasonable.

The *Mécanique analytique* which Lagrange had written in Berlin, was published in 1788. It had been approved for publication by a committee of the Académie des Sciences comprising of Laplace, Cousin, Legendre and Condorcet. Legendre acted as an editor for the work doing proof reading and other tasks. The *Mécanique analytique* summarised all the work done in the field of mechanics since the time of Newton and is notable for its use of the theory of differential equations. With this work Lagrange transformed mechanics into a branch of mathematical analysis. He wrote in the Preface:-

One will not find figures in this work. The methods that I expound require neither constructions, nor geometrical or mechanical arguments, but only algebraic operations, subject to a regular and uniform course.

Lagrange was made a member of the committee of the Académie des Sciences to standardise weights and measures in May 1790. They worked on the metric system and advocated a decimal base. Lagrange married for a second time in 1792, his wife being Renée-Françoise-Adélaide Le Monnier the daughter of one of his astronomer colleagues at the Académie des Sciences. He was certainly not unaffected by the political events. In 1793 the Reign of Terror commenced and the Académie des Sciences, along with the other learned societies, was suppressed on 8 August. The weights and measures commission was the only one allowed to continue and Lagrange became its chairman when others such as the chemist Lavoisier, Borda, Laplace, Coulomb, Brisson and Delambre were thrown off the commission.

In September 1793 a law was passed ordering the arrest of all foreigners born in enemy countries and all their property to be confiscated. Lavoisier intervened on behalf of Lagrange, who certainly fell under the terms of the law, and he was granted an exception. On 8 May 1794, after a trial that lasted less than a day, a revolutionary tribunal condemned Lavoisier, who had saved Lagrange from arrest, and 27 others to death. Lagrange said on the death of Lavoisier, who was guillotined on the afternoon of the day of his trial:-

It took only a moment to cause this head to fall and a hundred years will not suffice to produce its like.

The École Polytechnique was founded on 11 March 1794 and opened in December 1794 (although it was called the École Centrale des Travaux Publics for the first year of its existence). Lagrange was its first professor of analysis, appointed for the opening in 1794. In 1795 the École Normale was founded with the aim of training school teachers. Lagrange taught courses on elementary mathematics there. We mentioned above that Lagrange had a 'no teaching' clause written into his contract but the Revolution changed things and Lagrange was required to teach. However, he was not a good lecturer as Fourier, who attended his lectures at the École Normale in 1795 wrote:-

His voice is very feeble, at least in that he does not become heated; he has a very pronounced Italian accent and pronounces the s like z ... The students, of whom the majority are incapable of appreciating him, give him little welcome, but the professors make amends for it.

Similarly Bugge who attended his lectures at the École Polytechnique in 1799 wrote:-

... whatever this great man says, deserves the highest degree of consideration, but he is too abstract for youth.

Lagrange published two volumes of his calculus lectures. In 1797 he published the first theory of functions of a real variable with *Théorie des fonctions analytiques* although he failed to give enough attention to matters of convergence. He states that the aim of the work is to give:-

... the principles of the differential calculus, freed from all consideration of the infinitely small or vanishing quantities, of limits or fluxions, and reduced to the algebraic analysis of finite quantities.

Also he states:-

The ordinary operations of algebra suffice to resolve problems in the theory of curves.

Not everyone found Lagrange's approach to the calculus the best however, for example de Prony wrote in 1835:-

Lagrange's foundations of the calculus is assuredly a very interesting part of what one might call purely philosophical study: but when it is a case of making transcendental analysis an instrument of exploration for questions presented by astronomy, marine engineering, geodesy, and the different branches of science of the engineer, the consideration of the infinitely small leads to the aim in a manner which is more felicitous, more prompt, and more immediately adapted to the nature of the questions, and that is why the Leibnizian method has, in general, prevailed in French schools.

The second work of Lagrange on this topic *Leçons sur le calcul des fonctions* appeared in 1800.

Napoleon named Lagrange to the Legion of Honour and Count of the Empire in 1808. On 3 April 1813 he was awarded the Grand Croix of the Ordre Impérial de la Réunion. He died a week later.

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