

Stefan Banach

Born: 30 March 1892 in Kraków, Austria-Hungary (now Poland)

Died: 31 Aug 1945 in Lvov, (now Ukraine)

Stefan Banach's father was Stefan Greczek. The first thing to notice is that Banach was not his father's surname, but Banach was given his father's first name. Stefan Greczek was a tax official who was not married to Banach's mother who vanished from the scene after Stefan was baptised, when he was only four days old, and nothing more is known of her. The name given as Stefan's mother on his birth certificate is Katarzyna Banach. She is thought by some to have been the servant of Stefan's mother, while others claim that she was a laundress who took care of Stefan when he was very young. In later life Banach tried to find out who his mother was but his father refused to say anything except that he had been sworn to secrecy over her identity.

Stefan Greczek was born in a small village called Ostrowsko, some 50 km south of Kraków. It was to Ostrowsko, to his grandmother's home, that Banach was taken after his baptism. However, when Banach's grandmother took ill, Stefan Greczek arranged for his son to be brought up by Franciszka Płowa who lived in Kraków with her daughter Maria. Although Banach never went back to live with his grandmother, he did visit her frequently as he grew up. Maria's guardian was a French intellectual Juliusz Mien and he quickly recognised the talents that Banach had. Mien taught the young boy to speak French and in general gave him an appreciation for education.

Banach attended primary school in Kraków, leaving the school in 1902 to begin his secondary education at the Henryk Sienkiewicz Gymnasium No 4 in Kraków. By a fortunate coincidence, one of the students in Banach's class was Witold Wilkosz who himself went on to become a professor of mathematics. The school does not appear to have been a particularly good one and in 1906 Wilkosz left to move to a better Gymnasium. Banach, however, remained at Henryk Sienkiewicz Gymnasium No 4 although he maintained contact with Wilkosz.

During his first few years at the Gymnasium Banach achieved first class grades with mathematics and natural sciences being his best subjects. A fellow school pupil recalled Banach at this period in his life (see [3]):-

[Banach] was pleasant in dealings with his colleagues, but outside of mathematics he was not interested in anything. If he spoke at all, he would speak very rapidly, as rapidly as he thought mathematically. ... Wilkosz was a similar phenomenon. Between the two of them there was no mathematical problem that they could not speedily tackle. Also, while Banach was faster in mathematical problems, Wilkosz was phenomenally fast in solving problems in physics, which were of no interest to Banach.

The excellent grades of his early years gave way to poorer grades as he approached his final school examination. He passed this examination in 1910 but he failed to achieve a pass with distinction, an honour which went to about one quarter of the students. On leaving school Banach and Wilkosz both wanted to study mathematics, but both felt that nothing new could be discovered in mathematics so each chose to work in a subject other than mathematics. Banach chose to study engineering, Wilkosz chose oriental languages. That two such outstanding future mathematicians could make a decision for this reason must mean that there was nobody to properly advise them.

Banach's father had never given his son much support, but now once he left school he quite openly told Banach that he was now on his own. Banach left Kraków and went to Lvov where he enrolled in the Faculty of Engineering at Lvov Technical University. It is almost certain that Banach, without any financial support, had to support himself by tutoring. This must have occupied quite a lot of his time and when he graduated in 1914 he had taken longer to complete the course than was normal. He had returned to Kraków frequently during the

period of his studies in Lvov from 1910 to 1914. It is not entirely clear what Banach's plans were in 1914 but the outbreak of World War I in August, shortly after his graduation, saw Banach leave Lvov.

Lvov was, at the time Banach studied there, under Austrian control as it had been from the partition of Poland in 1772. In Banach's youth Poland, in some sense, did not exist and Russia controlled much of the country. Warsaw only had a Russian language university and was situated in what was named "Vistula Land". With the outbreak of World War I, the Russian troops occupied the city of Lvov. Banach was not physically fit for army service, having poor vision in his left eye. During the war he worked building roads but also spent time in Kraków where he earned money by teaching in the local schools. He also attended mathematics lectures at the Jagiellonian University in Kraków and, although this is not completely certain, it is believed that he attended Zaremba's lectures.

A chance event occurred in the spring of 1916 which was to have a major impact on Banach's life. Steinhaus, who had been undertaking military service, was about to take up a post at the Jan Kazimierz University in Lvov. However he was living in Kraków in the spring of 1916, waiting to take up the appointment. He would walk through the streets of Kraków in the evenings and, as he related in his memoirs:-

During one such walk I overheard the words "Lebesgue measure". I approached the park bench and introduced myself to the two young apprentices of mathematics. They told me they had another companion by the name of Witold Wilkosz, whom they extravagantly praised. The youngsters were Stefan Banach and Otto Nikodym. From then on we would meet on a regular basis, and ... we decided to establish a mathematical society.

Steinhaus told Banach of a problem which he was working on without success. After a few days Banach had the main idea for the required counterexample and Steinhaus and Banach wrote a joint paper, which they presented to Zaremba for publication. The war delayed publication but the paper, Banach's first, appeared in the *Bulletin of the Kraków Academy* in 1918. From the time that he produced these first results with Steinhaus, Banach started to produce important mathematics papers at a rapid rate. Of course it is impossible to say whether, without the chance meeting with Steinhaus, Banach would have followed the route of research in mathematics. It was also through Steinhaus that Banach met his future wife Lucja Braus. They were married in the mountain resort of Zakopane in 1920.

On Steinhaus's initiative, the Mathematical Society of Kraków was set up in 1919. Zaremba chaired the inaugural meeting and was elected as the first President of the Society. Banach lectured to the Society twice during 1919 and continued to produce top quality research papers. The Mathematical Society of Kraków went on to become the Polish Mathematical Society in 1920.

Banach was offered an assistantship to Lomnicki at Lvov Technical University in 1920. He lectured there in mathematics and submitted a dissertation for his doctorate under Lomnicki's supervision. This was, of course, not the standard route to a doctorate, for Banach had no university mathematics qualifications. However, an exception was made to allow him to submit *On Operations on Abstract Sets and their Application to Integral Equations*. This thesis [1]:-

... is sometimes said to mark the birth of functional analysis.

In 1922 the Jan Kazimierz University in Lvov awarded Banach his habilitation for a thesis on measure theory. The University Calendar for 1921-22 reports [3]:-

On 7 April 1922, by resolution of the Faculty Council, Dr Stefan Banach received his habilitation for a Docent of Mathematics degree. He was appointed Professor Extraordinary of that subject by decree of the Head of State issued on 22 July 1922.

In 1924 Banach was promoted to full professor and he spent the academic year 1924-25 in Paris. The years between the wars were extremely busy one for Banach. As well as continuing to produce a stream of important

papers, he wrote arithmetic, geometry and algebra texts for high schools. He also was very much involved with the publication of mathematics. In 1929, together with Steinhaus, he started a new journal *Studia Mathematica* and Banach and Steinhaus became the first editors. The editorial policy was:-

... to focus on research in functional analysis and related topics.

Another important publishing venture, begun in 1931, was a new series of *Mathematical Monographs*. These were set up under the editorship of Banach and Steinhaus from Lvov and Knaster, Kuratowski, Mazurkiewicz, and Sierpinski from Warsaw. The first volume in the series *Théorie des Opérations linéaires* was written by Banach and appeared in 1932. It was a French version of a volume he originally published in Polish in 1931 and quickly became a classic. In 1936 Banach gave a plenary address at the International Congress of Mathematicians in Oslo. In this address he described the work of the whole of the Lvov school, and he also spoke of the plans which they had to develop their ideas further.

Another important influence on Banach was the fact that Kuratowski was appointed to the Lvov Technical University in 1927 and worked there until 1934. Banach collaborated with Kuratowski and they wrote some joint papers during this period.

The way that Banach worked was unconventional. He liked to do mathematical with his colleagues in the cafés of Lvov. Ulam recalls in [4] frequent sessions in the Scottish Café:-

It was difficult to outlast or outdrink Banach during these sessions. We discussed problems proposed right there, often with no solution evident even after several hours of thinking. The next day Banach was likely to appear with several small sheets of paper containing outlines of proofs he had completed.

Andrzej Turowicz, also a professor of mathematics at the Kazimierz University in Lvov, also described Banach's style of working (see [3]):-

[Banach] would spend most of his days in cafés, not only in the company of others but also by himself. He liked the noise and the music. They did not prevent him from concentrating and thinking. There were cases when, after the cafés closed for the night, he would walk over to the railway station where the cafeteria was open around the clock. There, over a glass of beer, he would think about his problems.

In 1939, just before the start of World War II, Banach was elected as President of the Polish Mathematical Society. At the beginning of the war Soviet troops occupied Lvov. Banach had been on good terms with the Soviet mathematicians before the war started, visiting Moscow several times, and he was treated well by the new Soviet administration. He was allowed to continue to hold his chair at the university and he became the Dean of the Faculty of Science at the university, now renamed the Ivan Franko University. Banach's father came to Lvov fleeing from the German armies advancing towards Kraków. Life at this stage was little changed for Banach who continued his research, his textbook writing, lecturing and sessions in the cafés. Sobolev and Aleksandrov visited Banach in Lvov in 1940, while Banach attended conferences in the Soviet Union. He was in Kiev when Germany invaded the Soviet Union and he returned immediately to his family in Lvov.

The Nazi occupation of Lvov in June 1941 meant that Banach lived under very difficult conditions. He was arrested under suspicion of trafficking in German currency but released after a few weeks. He survived a period when Polish academics were murdered, his doctoral supervisor Lomnicki dying on the tragic night of 3 July 1941 when many massacres occurred. Towards the end of 1941 Banach worked feeding lice in German institute dealing with infectious diseases. Feeding lice was to be his life during the remainder of the Nazi occupation of Lvov up to July 1944. As soon as the Soviet troops retook Lvov Banach renewed his contacts. He met Sobolev outside Moscow but clearly he was by this time seriously ill. Sobolev, giving an address at a memorial conference for Banach, said of this meeting (see for example [3]):-

Despite heavy traces of the war years under German occupation, and despite the grave illness that was undercutting his strength, Banach's eyes were still lively. He remained the same sociable, cheerful, and extraordinarily well-meaning and charming Stefan Banach whom I had seen in Lvov before the war. That is how he remains in my memory: with a great sense of humour, an energetic human being, a beautiful soul, and a great talent.

Banach planned to go to Kraków after the war to take up the chair of mathematics at the Jagiellonian University but he died in Lvov in 1945 of lung cancer.

Banach founded modern functional analysis and made major contributions to the theory of topological vector spaces. In addition, he contributed to measure theory, integration, the theory of sets, and orthogonal series.

In his dissertation, written in 1920, he defined axiomatically what today is called a Banach space. The idea was introduced by others at about the same time, for example Wiener introduced the notion but did not develop the theory. The name 'Banach space' was coined by Fréchet. Banach algebras were also named after him.

A Banach space is a real or complex normed vector space that is complete as a metric space under the metric

$$d(x, y) = \|x - y\|$$

induced by the norm. The completeness is important as this means that Cauchy sequences in Banach spaces converge.

A Banach algebra is a Banach space where the norm satisfies

$$\|xy\| \leq \|x\| \cdot \|y\|$$

The importance of Banach's contribution is that he developed a systematic theory of functional analysis, where before there had only been isolated results which were later seen to fit into the new theory. The theory generalised the contributions made by Volterra, Fredholm and Hilbert on integral equations.

Banach proved a number of fundamental results on normed linear spaces, and many important theorems are today named after him. There is the Hahn-Banach theorem on the extension of continuous linear functionals, the Banach-Steinhaus theorem on bounded families of mappings, the Banach-Alaoglu theorem, the Banach fixed point theorem and the Banach-Tarski paradoxical decomposition of a ball.

The Banach-Tarski paradox appeared in a joint paper of the two mathematicians in 1926 in *Fundamenta Mathematicae* entitled *Sur la décomposition des ensembles de points en partiens respectivement congruent*. The puzzling paradox shows that a ball can be divided up into subsets which can be fitted together to make two balls each identical to the first. The axiom of choice is needed to define the decomposition and the fact that it is able to give such a non-intuitive result has made some mathematicians question the use of the axiom. The Banach-Tarski paradox was a major contribution to the work being done on axiomatic set theory around this period.

Banach's open mapping of 1929 also uses set-theoretic concepts, this time concepts introduced by Baire in his 1899 dissertation.

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